

# The Start Control of Heating System Based on Self-tuning

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**Abstract:** The problem of controlling the start-up of heating system is considered and the need for a self-tuning start control scheme is explained. The internal temperature change process of building is analyzed under the influence of heating source and external environment, and the change principle is found out. From the point of thermodynamics, the mathematical model of the process of intermittent heating is established on the basis of similitude principle. The self-tuning control strategy is introduced to finish the simulation and experiment of intermittent heating start, the results show that the self-tuning controller behaves well and has superior performance characteristics compared to two traditional methods.

**Key Words:** Self-tuning, Intermittent heating, Start-up

## 1 Introduction

Building energy consumption accounts for nearly 45% of China's energy consumption today, and the energy cost of heating system takes about 20% of the whole building energy consumption in western China<sup>[1]</sup>. Therefore, energy saving modification of heating system will lead to good economic and social benefits.

According to the difference of the using characters of buildings, the heating system is divided into two categories: the continuous heating system and the intermittent heating system<sup>[2]</sup>. Heating system runs by the intermittent operation mode as Fig. 1

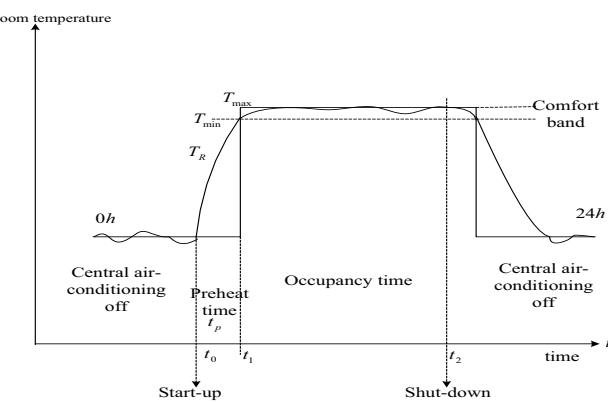


Fig. 1: Intermittent operation of the heating system

Owe to the thermal inertia of the heating system and that of the buildings, a preheat time  $t_p$  is necessary so that the required temperature is reached at the moment of occupation. The performance of any start-up controller will depend on the accurate estimation of the preheat time. Overestimation will cause unnecessary energy wastage, while underestimation will lead to loss of comfort conditions at the start of occupancy. Therefore, it is necessary to compute the accurate preheat time and control the heating system in order to satisfy the comfort conditions and save energy.

Because of the unknown characters of building and heating system, it is difficult to realize the optimum start-up control of heating system with traditional control schemes.

Even if traditional control schemes are applied to control the heating system, the prediction errors of the preheat time are very big and the influence of time delay can be not overcome. The self-tuning scheme can identify system parameters online according to the change of characters of the plant and environmental disturbance. It is suitable for the control field with slow changes of model and disturbance. Therefore, the self-tuning control scheme is introduced to control the start-up of the heating system. It can save energy by forecasting accurately the preheat time based on the temperature prediction model whose parameters are identified by the one-step-ahead minimum variance algorithm. This method is suitable for most intermittent heating system because it avoids getting the characters of heating system and heating zone.

## 2 The Preheat Time Model of Building

The differential equation governing the time variation in the room temperature  $T_r$  is<sup>[3-5]</sup>,

$$\tau_1 \frac{dT_r(t)}{dt} + T_r(t) = \tau_1 R_1 \frac{dQ_h(t)}{dt} + (R_1 + R_2)Q_h(t) + T_o(t) \quad (1)$$

where  $Q_h$  is heat output of the heating system. The time constant of the structure  $\tau_1 = C_b R_2$  where  $C_b$  is the equivalent thermal capacity of the structure located between two thermal resistances;  $R_1$  to the inside and  $R_2$  to the outside of the building.  $T_o(t)$  is the external air temperature.

It is easy to get the solution of equation (1). The solution is described with equation (2).

$$T_r(t) = Q_h \{R_1 + R_2(1 - e^{-\frac{t}{\tau_1}})\} + T_b(t) \quad (2)$$

where  $T_b$  is the bulk temperature of the structure. The first expression describes the component of the room temperature variation due to start-up of the heating system, the second expression describes the component due to changes in the external air temperature. The bulk temperature can be computed by equation (3),

$$\tau_1 \frac{dT_b(t)}{dt} + T_b(t) = T_o(t) \quad (3)$$

The ideal preheat control will start at  $t=0$ , and  $T_r(t) = T_d$  when  $t=t_p$ , where  $T_d$  is the desired room temperature. Because the bulk temperature varies slowly in practice, the preheat time can be got at  $t=0$  from following equation (4),

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$$t_p(0) = \tau_1 \ln \left\{ \frac{R_2 Q_h}{Q_h (R_1 + R_2) - T_d + T_b(0)} \right\} \quad (4)$$

Because equation (4) is non-linear in the parameters, it is unsuitable to incorporate in the self-tuning control scheme. However, the preheat time is much shorter than the time constants of the building structure in practice, that is to say, when  $e^{-t_p/\tau_1} \approx 1 - t_p/\tau_1$ , equation (4) can be simplified,

$$t_p(0) = \frac{\tau_1}{Q_h R_2} \{T_d - Q_h R_1 - T_b(0)\}$$

$$\text{or } t_p(0) = k_1 + k_2 T_b(0) \quad (5)$$

considering the heat output must take some time to get steady after heating system works, and the above factor and the thermal inertia of room affect the preheat, a more accurate representation is obtained<sup>[5]</sup>,

$$t_p(0) = \left\{ \frac{T_d - Q_h / H_i - T_b(0)}{Q_h \sqrt{4R_e / \Pi C_e}} \right\}^2$$

$$\text{or } t_p(0) = k_3 + k_4 T_b(0) + k_5 T_b^2(0) \quad (6)$$

where  $H_i$  is the internal air film resistance,  $R_e$  is the equivalent bulk thermal resistance through the structure, and  $C_e$  is the equivalent bulk thermal capacity of the structure.

Equation (6) includes the equation (5), the linear model as a special case.

So far, the derivations have ignored the possibility of heat flows to or from the structure which are not taken into account in the calculation of bulk temperature by a single point measurement of the external air temperature, for example, solar heat gain, wind losses or heat flows to or from adjoining rooms. Since these unmeasured disturbances might be expected to show degree of correlation from day to day, a simple first-order moving-average noise process with non-zero mean can be included in equation (6) to describe their influence on preheat time. Thus, rewriting equation (6) in discrete time form, the preheat time at the  $n$ th sampling instant is given by the difference equation<sup>[5]</sup>,

$$t_p(n) = k_6 + k_7 T_b(n) + k_8 T_b(n)^2 + e(n) + k_9 e(n-1) \quad (7)$$

where  $e(n)$  is taken from a zero mean independent noise sequence.

### 3 The Preheat Time Prediction Based on the Self-tuning Control

#### 3.1 The Self-tuning Control

The optimum start-up control of heating system must be on the basis of analyzing the heating system. By this optimum start-up control, the energy consumption reduces to the minimum. However, Traditional optimum control methods involve building structure and environment factors that are non-linear and time-varying so that the analysis on optimum start-up control of heating system is complicated and difficult, and the temperature evolution is delay and slow along with random disturbance, therefore, the self-tuning scheme can be used to realize the optimum start-up control of heating system and save energy.

The important character of self-tuning control is that there is an on-line recognition unit that is a recursive estimator of model parameters of the controlled object<sup>[6-7]</sup>. The typical control system structure of self-tuning is showed in Fig.2.

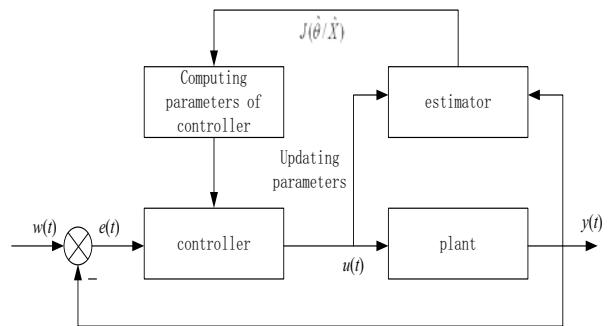


Fig.2: The control system structure of self-tuning

The control system of self-tuning consists of an adjustable controller, controlled object, estimator of parameters or states and computing unit of parameters. Estimator of parameters or states estimates the current parameter  $\theta$  or state variable  $x$  of controlled object with system identification method according to the observation sequence of the system input  $u(t)$  and output  $y(t)$ , and the estimated value of parameter  $\hat{\theta}$  or state variable  $\hat{x}$  is sent to the computing unit of parameters that is actually a set of auto-tuning algorithm. It uses performance index function  $J$  and the  $\hat{\theta}$  or  $\hat{x}$  to realize the auto-tuning and update the controller parameters. The new control goes on so that the system always is in optimal or sub-optimal working conditions.

In this paper, the minimum variance control is adopted, and the control law is designed according to the minimum output variance. Because the heating system has the character of slowly-time-varying parameters, the weight of the old data must be discounted so that the changes of dynamic process can be followed, the weighted recursive least squares estimation algorithm is applied to estimate directly the controller parameters.

According to the minimum variance control theorem, the control target is to minimize the errors variance  $J = E\{[y(t+d) - y_r(t+d)]^2\}$  between actual output  $y(t+d)$  and desired output  $y_r(t+d)$ . the minimum variance control law is showed by equation (8) ,

$$F(q^{-1})u(t) = y_r(t+d) + [C(q^{-1}) - 1]y^*(t+d|t) - G(q^{-1})y(t) \quad (8)$$

where the Hurwitz polynomial  $C(q^{-1})$  is the parameter of the mathematical model of the controlled object.

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + C(q^{-1})\xi(t) \quad (9)$$

The formulation of the weighted recursive least squares estimation algorithm is showed by equation (10)<sup>[8]</sup>,

$$\begin{cases} \hat{\theta}(n) = \hat{\theta}(n-1) + K(n)\varepsilon(n) \\ P(n) = \frac{1}{\lambda} \left\{ P(n-1) - \frac{P[(n-1)]\varphi(n)\varphi^T(n)P(n-1)}{\lambda + \varphi^T(n)P(n-1)\varphi(n)} \right\} \\ K(n) = \frac{P(n-1)\varphi(n)\varepsilon(n)}{\lambda + \varphi^T(n)P(n-1)\varphi(n)} \\ \lambda = 0.995 \sim 0.999 \end{cases} \quad (10)$$

### 3.2 The Preheat Time Prediction

If the parameters in equation (7) are known, the preheat time can be estimated by the minimum variance predictor. Therefore, the estimated value of preheat time is computed by equation (11)<sup>[3][9]</sup>.

$$\hat{t}_p(n) = k_6 + k_7 T_b(n) + k_8 T_b(n)^2 + k_9 \varepsilon(n-1) \quad (11)$$

where  $\varepsilon(n-1) = t_p(n-1) - \hat{t}_p(n-1)$ ,

$$\hat{T}_b(n) = \alpha \hat{T}_b(n-1) + \beta T_o(n) + \gamma T_r(n) \quad (12)$$

According to the reference 3,  $\alpha=0.8$ ,  $\beta=(1-\alpha)f$ ,  $\gamma=(1-\alpha)(1-\beta)$ . The equation (11) rewrites with vector form and the equation (13) is obtained.

$$t_p(n) = \phi^T(n)\theta + \varepsilon(n) \quad (13)$$

where  $\phi^T(n) = [\varepsilon(n-1) \ T_b(n) \ T_b^2(n) \ 1]$ ;

$$\theta^T = [k_9 \ k_7 \ k_8 \ k_6]$$

When these parameters are estimated by the minimum variance self-tuning predictor, they are revised automatically in each sampling interval to reduce the variance of the prediction error.

Combining the weighted recursive least squares estimation algorithm and the equation (13), the preheat prediction is then given by the equation (14),

$$\hat{t}_p(n) = \phi^T(n)\hat{\theta}(n-1) \quad (14)$$

### 4 The Simulation and Experiment of the Optimal Preheat Time Prediction of Building

#### 4.1 The Algorithm of the Optimal Preheat Time Prediction

The simulation is finished with the soft Matlab. In the simulation, the algorithm of start based on the self-tuning as equation (10) and (14) is realized by Matlab program. The algorithm is showed as the follow.

- 1) get the self-tuning control law form the prediction model
- $\hat{t}_p(n) = k_6 + k_7 T_b(n) + k_8 T_b(n)^2 + k_9 \varepsilon(n-1);$
- 2) set the initial values as  $T_b(0)$ ,  $t_p(0)$ ,  $\theta(0)$ ,  $P(0)$ ,  $\phi^T(0)$ ;
- 3) sample and read the new observation data as  $T_o(k)$ ,  $t_p(k)$ ; Let  $\alpha=0.8$ ,  $\beta=(1-\alpha)f$ ,  $\gamma=(1-\alpha)(1-\beta)$ , and  $T_b(k) = \alpha \hat{T}_b(k-1) + \beta T_o(k) + \gamma T_r(k)$
- 4) format the new observation vector  $K(k)$ ,  $\phi^T(k)$ ;
- 5) compute the  $\hat{\theta}(k)$   $P(k)$  with weighed RLS;
- 6) compute the  $\hat{t}_p(k)$  with equation (14) ;
- 7) start timer and control;
- 8)  $k=k+1$ , continue compute from step 3 to step 8.

#### 4.2 The Simulation and Experiment of the Optimal Preheat Time Prediction of Building

The simulation samples consist of the data of 150 days that were recorded from November to March, which includes outside air temperature and practical preheat time.

The data of 150 days are input and are divided into 25 sets of sample data in simulation. They are used to predicate the preheat time.

The initial value of  $\theta$  is assigned,  $\theta=[k_9, k_7, k_8, k_6]=[0.0000 \ 0.0000 \ 0.0000 \ 1.5000]$ , and the weighting factor in weighted RLS is equal to 0.995. When the simulation is finished,  $\theta=[0.032 \ 0.073 \ 0.162 \ 0.0092]$ . The estimated parameters at every six days are showed as Fig.3, Fig. 4, Fig.5 and Fig.6, and the prediction errors are showed as Fig.7.

Through accuracy statistics and analysis, the average value of the preheat time errors is equal to 0, and the preheat time errors distribute from -0.24 hours to 0.16 hours, the days are 135 days whose preheat time errors arrange from -0.12 hours to 0.12 hours.

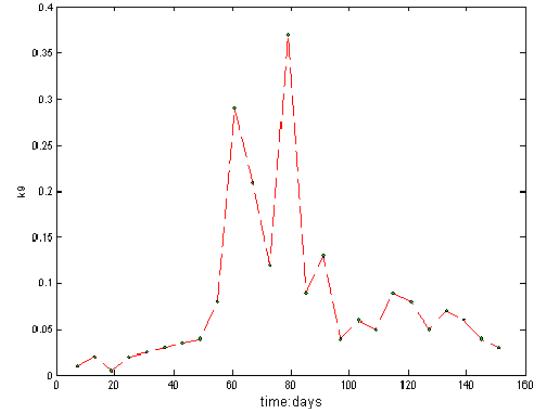


Fig.3: the parameter k9 of simulation

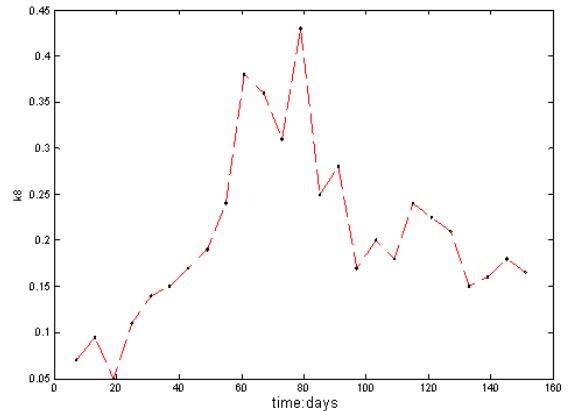


Fig.4: the parameter k8 of simulation

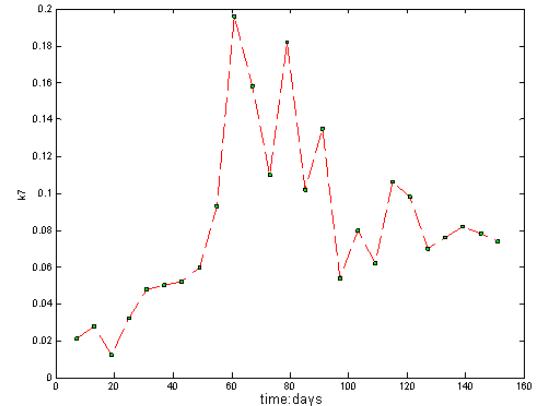


Fig.5: the parameter k7 of simulation

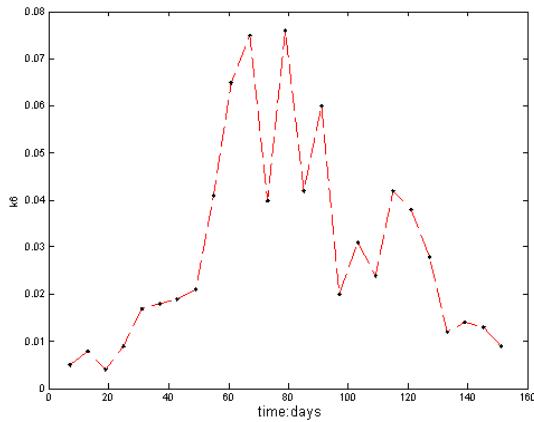


Fig.6: the parameter  $k_6$  of simulation

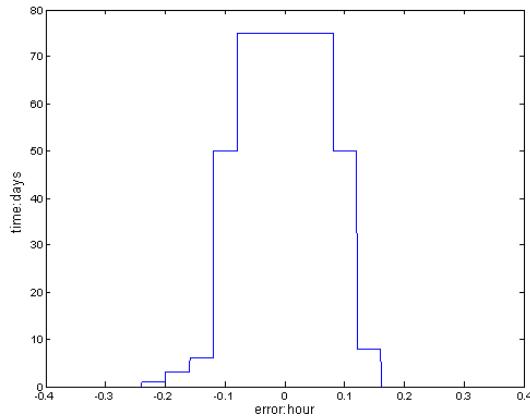


Fig.7: the prediction error statistics of simulation

Based on the above self-tuning control algorithm, the controller is realized with single chip, and it is applied in heating system. The experiment result of 20 days is recoded and showed in Fig. 8. From the Fig.8, we know the preheat errors of the first and the second day are not enough little and other errors are very little that distribute between -0.13 hours to 0.14 hours.

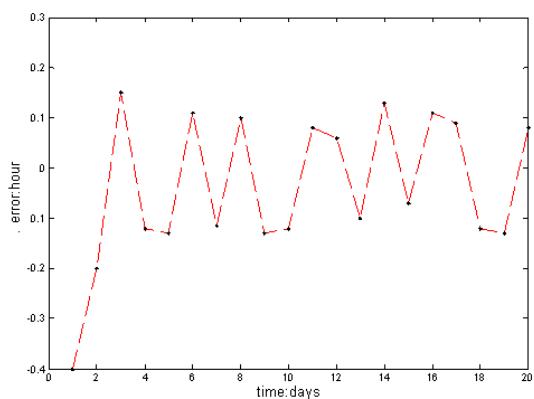


Fig.8: the prediction error statistics of experiment

The experiment shows that the self-tuning controller has good character of prediction and control even if the initial parameters of preheat model is not accurate and the thermal performance of building and heating system are unknown.

## 5 Conclusions

A self-tuning control scheme is introduced to control the start-up of heating system. It has the advantage of knowing no thermal characters of the heating system and the building. The results of simulation and experiment are very acceptable. Comparing with two traditional methods in which one takes the biggest practical preheat time in recent years as constant preheat time, the other adjusts the preheat time by the skilled operator in the light of the everyday weather during the heating period, the method in this paper not only saves energy, but also has a high level of automation. The self-tuning start-up controller can be applied widely in heating system.

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